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Geometric Analysis of Low-Earth-Orbit Satellite Communication Systems: Covering Functions

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A new approximate method of coverage analysis for real-time communication systems based on low-Earth-orbit satellite constellations in circular orbits has been developed. A set of covering functions that describes coverage statistics vs geographical latitude is introduced. The *i*th-order covering function value is a geocentric angle of the smallest circle that encloses the subsatellite points of no less than *i* satellites at any time. New statistical parameters of coverage based on the first-order covering function (guaranteed estimations of the summary visibility period in which at least one satellite has to be seen from a user, of a joint visibility period of two adjacent satellites, and a maximal waiting time of a next visibility period with a guaranteed estimation of the summary visibility period) are described. The maximal distance between two users with a joint visibility of a satellite can be determined using this function. Methods for the computation of local and regional service areas for a network topology using ground stations are proposed. Numerical examples for the IRIDIUM® and GLOBALSTAR constellations are presented.

Nomenclature

h	= orbit altitude, km
R_e	= spherical radius of the Earth, 6378 km
T	= orbital period
t	= time
Z_{GSi}	= service area of ith ground station
α	= elevation angle, deg; see Fig. 1
Δt	= visibility period of satellite
$\Delta t_{\max}$	= maximal visibility period of satellite; Eq. (2)
Δt_{1W}	= maximal waiting time of a next visibility period
δt_{12}, δt_{23}	= joint visibility period of two and three satellites
Θ	= coverage angle; see Fig. 1
Θ_i	= ith-order covering function; Eq. (4)
λ	= longitude
μ	= Earth's gravitational constant, 398,600 km^3/s^2
φ	= latitude
Ψ	= azimuth

Introduction

CURRENTLY, there is great interest in providing worldwide communication services to mobile or other users via networks of low-Earth-orbit (LEO) satellites.[1,2] The design of satellite communication systems relies on an analysis of a number of mutually linked factors, such as type of service, optimal satellite constellation, minimal elevation angle, extent and distribution of the expected user population, class of satellites, types of mobile terminals and ground stations (GSs), etc. Comparison of different alternatives is difficult due to complex interactions between these elements.

In studying mathematical models of these complex systems, one often faces a tradeoff between precision of model predictions and ability to obtain detailed information needed to build the model. In the present paper, a new approximate method of coverage analysis for real-time communication systems based on LEO satellite constellations in circular orbits was formulated. The rationale of this method is to obtain a better understanding of the nature of LEO satellite constellations. This knowledge would be useful for first-hand design of communication systems. Also, approximate methods often serve a useful role as a starting of reference solutions for more accurate approaches.

Real-time communication systems require a satellite constellation that can provide continuous global or zonal coverage. This problem for satellite constellations in circular orbits has been examined by many authors since about 1960.[3–18]

It is well known that the coverage statistics for satellite constellations depend on geographical latitudes. Currently, statistical parameters of coverage based on an elevation angle (an average number of satellites with an elevation angle greater than a specified value, a time percentage of periods in which the elevation angle is greater than a specified value, etc.) are more often used. As a continuous measure of coverage statistics vs latitude, we propose new descriptive functions, here called *covering functions.* The ith order covering function value is a geocentric angle of the smallest circle on the Earth's surface that encloses subsatellite points of no less than i satellites at any time. In a sense, these functions describe satellite densities as functions of latitude. There are many possible applications of such functions. The first is a standard application of the computation of a guaranteed visibility period of a satellite, a joint visibility period of two satellites from users or GSs, a maximal distance between two users with joint visibility of a satellite, etc. The second is determination of service areas for user–satellite–user (U–S–U) and user–satellite–GS–satellite–user (U–S–GS–S–U) link chains. The third is a calculating of regional or global service areas based on a GS network.

Note that there are some similarities between the covering functions and geometric parameters of satellite pairs and triads for satellite constellations introduced by Walker[4,9] and Ballard[8] (a largest value of the radius of the circumcircle of adjacent satellites and a range arc from midpoint of the spherical triangle formed by joining three subsatellite points). However, there is a fundamental difference. These parameters describe the worst-case observer on the Earth's surface independent of its latitude. By contrast, the new functions are functions of latitude, which determine the worst-case observer located at any point with this latitude. Although the cited parameters are used, as a rule, for a computation of optimal satellite constellations, we believe that the covering functions are more suitable for an analysis of specified satellite constellations.

LEO Satellite Communication Systems

Geometrical Preliminaries

Figure 1 shows a typical satellite coverage of the Earth for an observer (a user or a GS). The satellite is located at an orbital height h; the projection of the footprint onto the Earth's surface defines a circle of coverage of angle θ. The coverage angle can be expressed as a function of the orbital altitude h and a minimal elevation angle α within the coverage area as follows:

$$\theta = \cos^{-1}\{[R_e/(R_e + h)]\cos\alpha\} - \alpha \tag{1}$$

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Table 1 Link types

Designation	Link chain	Figure
U–S–U	User–satellite–user	2a
S–GS–S	User–satellite–GS–satellite–user	2b
GS–S–GS	User–satellite–GS–satellite–GS–··· –GS–satellite–user	2c

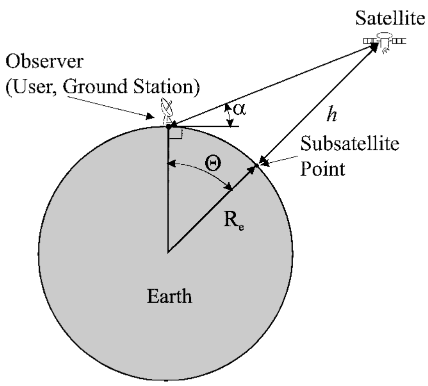


Fig. 1 Observer-to-satellite geometry.

The user speed (relative to the Earth) and the Earth rotation velocity (the maximum for the near-equatorial areas is about 0.46 km/s) are negligible as compared to the LEO satellite velocity ($\sim$7.5 km/s). In this case, the users and GSs can be conceived of as nonmoving points in an inertial frame during the visibility period of a satellite.

The maximal satellite visibility period corresponds to a satellite groundtrack that passes through the observer:

$$\Delta t_{\max} = (\theta/\pi)T \tag{2}$$

where

$$T = 2\pi\sqrt{(R_e + h)^3/\mu} \tag{3}$$

is the orbital period. The time $\Delta t_{\max}$ is a function of θ and will be indicated as $\Delta t(\theta)$.

Satellite Constellations

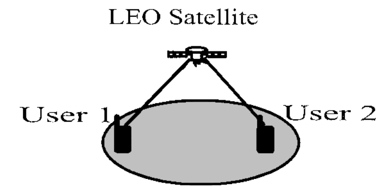
This paper deals with two well-known LEO constellations: the IRIDIUM® constellation[19] and GLOBALSTAR constellation.[20] The IRIDIUM system uses intersatellite links to provide communication services.[19] The GLOBALSTAR system uses transmitting radio signals via the satellite network between GSs and users.

The IRIDIUM constellation consists of 66 operational satellites. These satellites are arranged in six orbital planes 780 km above Earth, each containing 11 satellites. Their orbits are inclined at an angle of 86.4 deg. The satellites form a constellation that provides single global coverage for a minimal elevation angle of 8.2 deg. The IRIDIUM constellation[19] belongs to a general class of near-polar satellite constellations,[18] which is a generalization of the Adams and Rider family of the polar-phased constellations.[12]

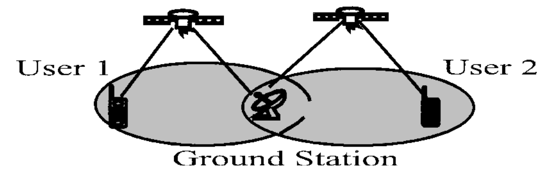
The GLOBALSTAR constellation consists of 48 satellites in eight planes with six equally spaced satellites in each plane with an inclination of 52 deg and an altitude of 1390 km. The constellation is designed for 100% single coverage between ±70° latitude and double coverage between 25° and 50° latitudes for a minimal elevation angle of 10 deg. This constellation belongs to symmetrical constellations of the Walker type.[4,9] In the following coverage analysis, we will be using only nominal parameters of the constellations because the formation keeping of a long-term structure of a LEO constellation can be realized with a high degree of accuracy.[21,22]

Network Topology

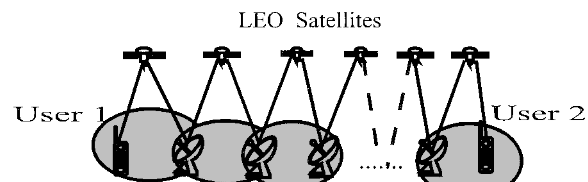
Recently, various aspects of communication using LEO satellite constellations have been investigated.[1,23–26] We consider network topologies using the satellites as relays between users and/or GSs. Link types are given in Table 1 and shown in Fig. 2.



a) Local link of type U–S–U



b) Link of type S–GS–S



c) Regional or global links of type GS–S–GS

Fig. 2 Link types.

Covering Functions

Definition of Covering Functions

Consider an observer on the Earth's surface and a satellite constellation. At any given t, we can compute the Earth-central angle ϑ_j between the observer (with latitude φ and longitude λ) and subsatellite point of the jth satellite of the constellation. It is evident that the minimal angle $\min(\vartheta_j)$ to the nearest satellite can be changed from zero, for example, it is a groundtrack that passes through the observer, to a maximal value. As a rule, the typical lifetime of a communication satellite constellation is from several months to several years. Long-term evolution of satellite constellations is a complex motion about the Earth's surface. On one hand, the observer rotates with the Earth, and on the other hand, the orbit of each satellite precesses about an inertial frame. For a long-term interval, we can assume that the maximal value of $\min(\vartheta_j)$ refers equally to an arbitrary site with latitude φ regardless of longitude. Similar extremal angles can be computed for two or more nearest satellites.

We define the covering functions for LEO satellite constellations as follows. In a geometric sense, the ith-order covering function value is the geocentric angle of the smallest circle that encloses subsatellite points for no fewer than i satellites at any time. Thus, we can write

$$\theta_1(\varphi) = \max_{\substack{t\to\infty \\ 0\le\lambda<2\pi}} \left\{ \min_j [\vartheta_j(\varphi,\lambda,t)] \right\}$$

$$\theta_2(\varphi) = \max_{\substack{t\to\infty \\ 0\le\lambda<2\pi}} \left\{ \min_{k\ne j} [\vartheta_k(\varphi,\lambda,t)] \right\}$$

$$\cdots\cdots\cdots\cdots$$

$$\theta_n(\varphi) = \max_{\substack{t\to\infty \\ 0\le\lambda<2\pi}} \left\{ \min_{l\ne j,k,\ldots} [\vartheta_l(\varphi,\lambda,t)] \right\}$$

$$\cdots\cdots\cdots\cdots$$

$$\theta_N(\varphi) \equiv \pi \tag{4}$$

where $j, k, l, \ldots = 1, 2, \ldots, N$ and N is the total number of satellites.

The constraint relation of the ith-order continuous coverage based on the ith-order covering function can be written as

$$\theta \ge \max_{\varphi\subset F} \theta_i(\varphi) \tag{5}$$

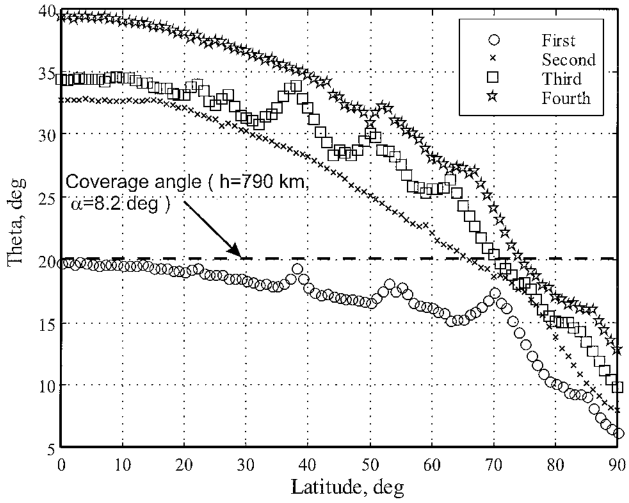


Fig. 3 Covering functions of the IRIDIUM constellation.

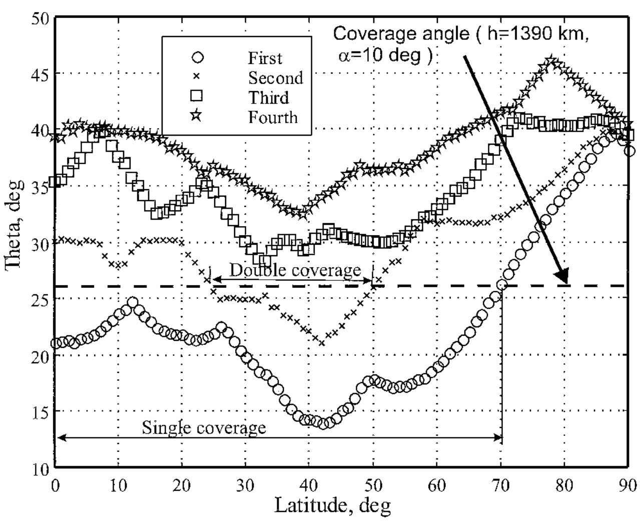


Fig. 4 Covering functions of the GLOBALSTAR constellation.

where F is a required interval of latitudes. That is, visibility is achieved when the equation is satisfied.

It is also easy to show that the ith-order covering function $\theta_i(\varphi)$ is in one-to-one correspondence with a maximal elevation angle required to guarantee continuous visibility (from any point with latitude φ) of at least i satellites. The inverse transformation can be obtained from Eq. (1).

Examples of Covering Functions

Examples of the covering functions $\theta_1(\varphi)$, $\theta_2(\varphi)$, $\theta_3(\varphi)$, and $\theta_4(\varphi)$ for the IRIDIUM and GLOBALSTAR constellations are shown in Figs. 3 and 4, respectively.

For a computation of the covering functions, a Monte Carlo method or a time-step simulation can be used. The computational effort is comparable with a standard statistical simulation of satellite constellations. In the presented numerical examples, a Monte Carlo simulation with 1000 instantaneous possible positions of the satellite constellations relative to a set user was used. For each position, a random set of users from the equator to the pole (with latitude step of 1° and a uniform distribution in longitude) was computed.

In the following computations and analysis, only tabulated or interpolated data for the covering functions are used.

Estimations Based on Covering Functions

In this section, estimations of statistical parameters of coverage for LEO constellations are presented. All of these estimations are guaranteed times referring equally to any observer with the same latitude φ.

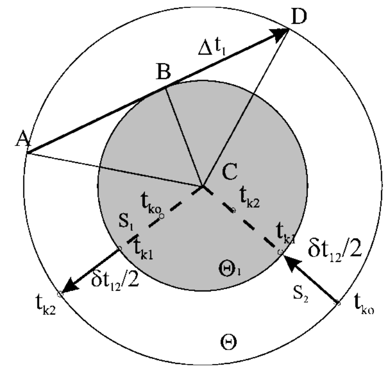


Fig. 5 Geometry of the first covering function; times are t_{k0}, start of visibility of S_2 satellite; t_{k1}, out of satellite S_1 from circle Θ_1 and intersection of circle Θ_1 by S_2 satellite; and t_{k2}, end of visibility of S_1 satellite.

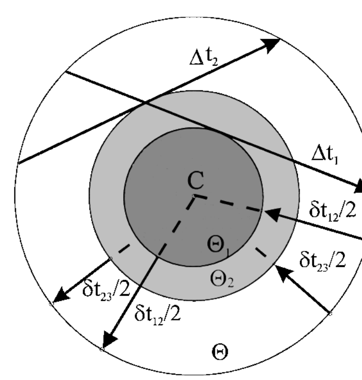


Fig. 6 Geometry of the second covering function.

Properties of the First Covering Function

Figure 5 shows a geometric interpretation. The two circles are on the surface of the Earth and have radii defined by Earth-central angles θ_1 and θ. We assume that $\theta_1(\varphi) < \theta$ (Fig. 5).

Property 1: The first covering function (4) guarantees that a circle θ_1 encloses the subsatellite point of at least one satellite. A boundary case is a groundtrack that passes tangent to the circle θ_1. Then, applying the law of cosines to the spherical triangle *BDC* (Fig. 5) yields

$$\cos\theta = \cos\theta_1 \cos BD \tag{6}$$

Therefore, at any time there is at least one satellite with a summary visibility period no less than

$$\Delta t_1(\varphi) = \frac{T}{\pi}\cos^{-1}\left(\frac{\cos\theta}{\cos\theta_1(\varphi)}\right) \le \Delta t_{\max} \tag{7}$$

Property 2: There is a pair of two adjacent satellites (each with visibility periods no less that Δt_1) with a joint visibility period no less than

$$\delta t_{12}(\varphi) = \{[\theta - \theta_1(\varphi)]/\pi\}T \tag{8}$$

The boundary case is a combination of two groundtracks each with the minimal time of motion between circles θ_1 and θ denoted $\delta t_{12}/2$ (see Fig. 5; we assume that the groundtracks are straight lines because $\theta_1 \ll \pi$, and $\theta \ll \pi$). By this means, the summarized joint visibility period of the two satellites is δt_{12}. Clearly, $\delta t_{12} < \Delta t_1$.

Property 3: The maximal waiting time for the next visibility period of a satellite with the duration no less than Δt_1 is

$$\Delta t_{1w}(\varphi) = \Delta t_{\max}(\theta) - \delta t_{12}(\varphi) \tag{9}$$

measured from when the first satellite becomes visible.

The case with the longest waiting time is a combination of the maximal visibility period $\Delta t_{\max}(\theta)$ and minimal joint visibility period of two satellites δt_{12}.

Estimations Based on the Second Covering Function

Similar estimations can be defined using the second covering function $\theta_2(\varphi)$. We assume that $\theta_1(\varphi) \le \theta_2(\varphi) < \theta$ (Fig. 6). The proofs are omitted.

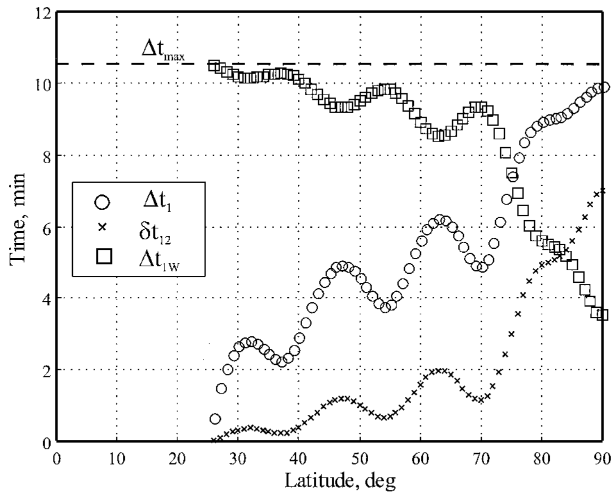


Fig. 7 Statictical parameters of coverage for the IRIDIUM constellation.

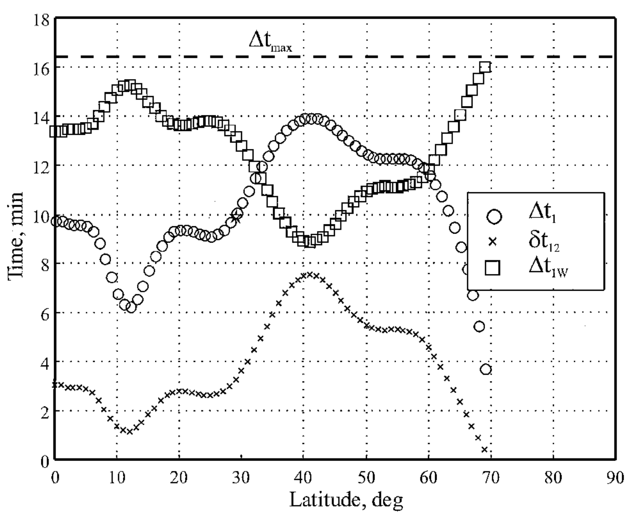


Fig. 8 Statictical parameters of coverage for the GLOBALSTAR constellation.

Property 5: At any time there are at least two satellites each with a summary visibility period no less than Δt_1 and

$$\Delta t_2(\varphi) = \frac{T}{\pi}\cos^{-1}\left(\frac{\cos\theta}{\cos\theta_2(\varphi)}\right) \leq \Delta t_1(\varphi) < \Delta t_{\max} \tag{10}$$

Property 6: There is a triad of satellites with the joint visibility periods no less than

$$\delta t_{23}(\varphi) = \{[\theta - \theta_2(\varphi)]/\pi\}T \tag{11}$$

Examples of Statistical Parameters of Coverage

Estimations of statistical parameters of coverage Δt_1, δt_{12}, and Δt_{1W} for the IRIDIUM and GLOBALSTAR constellations are plotted in Figs. 7 and 8, respectively.

Service Areas

Service areas define regions of the world in which a communication system will be used. They are expressed in terms of geographical areas.

Local Service Areas of Type U–S–U

At any time a local link of type U–S–U is possible between two users located in the footprint of the same satellite if $\theta > \theta_1$. As an example, it is possible in the case of at least single coverage. However, the maximal distance allowable between the two users depends on the first covering function $\theta_1(\varphi)$ in a neighborhood of the users' latitudes and the azimuth of the connecting line.

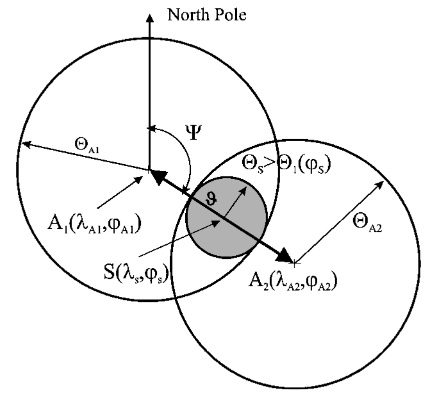


Fig. 9 Geometry of local link.

Suppose that the two users are located at points A_1 and A_2 with coordinates (λ_1, φ_1) and (λ_2, φ_2), respectively (see Fig. 9). Let $\theta_{\hat{A}1}$ and $\theta_{\hat{A}2}$ denote coverage circles of the users. If these circles are nonoverlapping, then the link between the users is impossible. Denote θ_S as the largest circle radius inscribed on the overlapping area of the coverage circles. Let (λ_S, φ_S) be the coordinates of the center of this circle. If $\theta_S < \theta_1(\varphi_S)$, then the link is possible when the two users stay in the footprint of a satellite. Finally, if

$$\theta_S \geq \theta_1(\varphi) \tag{12}$$

then, at any time, there is guaranteed visibility to at least one satellite from the two users. In this case, the links between them can always be established. The circle θ_S may be considered as a coverage circle of a user. Thus, the estimation of statistical parameters of coverage (Δt_1, δt_{12}, and Δt_{1W}) can be applied to joint visibility of the two users, and they take on a new meaning. As an example, the time Δt_1 is a guaranteed in-view period of one satellite from the two users and the δt_{12} is a joint visibility period of another pair of satellites from the two users. On the other hand, we can obtain an estimation of the maximal in-view period $\Delta t_{\max}(\theta_S)$. However, the overlapping area of the coverage circles is greater than the inscribed circle θ_S. Therefore, the estimation implies a true maximal in-view period no less than $\Delta t_{\max}(\theta_S)$. The values Δt_1, δt_{12}, Δt_{1W}, and $\Delta t_{\max}(\theta_s)$ depend on the distance and the azimuth of the connecting line between the two users. In the boundary points of the service area, that is, $\theta_S \to \theta_1$, then $\Delta t_1 \to 0$, $\delta t_{12} \to 0$, $\Delta t_{1W} \to 0$, and $\Delta t_{\max} \to \Delta t_{\max}(\theta_1)$.

Expression (12) is an implicit equation that defines the biggest service area with continuous links. The coordinates of a maximal remote user A_2 depend on the first covering function $\theta_1(\varphi_S)$ and an azimuth of the connecting line Ψ, and in turn the latitude φ_S depends on the coordinates A_2. For a solution of this problem with given values of λ_1, φ_1, Ψ, $\theta_{\hat{A}1}$, and $\theta_{\hat{A}2}$ and the single covering function $\theta_1(\varphi)$, an iterative method can be used. At the first iteration, we assume that $\varphi_S^{(0)} \approx \varphi_1$. The following computational algorithm has been proposed (based on equations for latitude and longitude that are given in the Appendix).

1) Compute the Earth-central angle between points S and A_1:

$$\vartheta_S^{(K+1)} = \theta_{A1} - \theta_1\left(\varphi_S^{(K)}\right) \tag{13}$$

where $K = 0, 1, 2, \ldots,$ is the iteration number.

2) Compute the latitude φ_S:

$$\varphi_S^{(K+1)} = \sin^{-1}\left[\cos\left(\vartheta_S^{(K+1)}\right)\sin\varphi_1 + \sin\left(\vartheta_S^{(K+1)}\right)\cos\varphi_1\cos\Psi\right] \tag{14}$$

3) Repeat steps 1 and 2 until a preestablished accuracy of latitude φ_S is achieved.

4) Determine the Earth-central angle between points A_1 and A_2:

$$\vartheta = \theta_{A1} + \theta_{A2} - 2\theta_1(\varphi_S) \tag{15}$$

5) Compute the latitude and longitude of the point A_2:

$$\varphi_2 = \sin^{-1}(\cos\vartheta\sin\varphi_1 + \sin\vartheta\cos\varphi_1\cos\psi) \tag{16a}$$

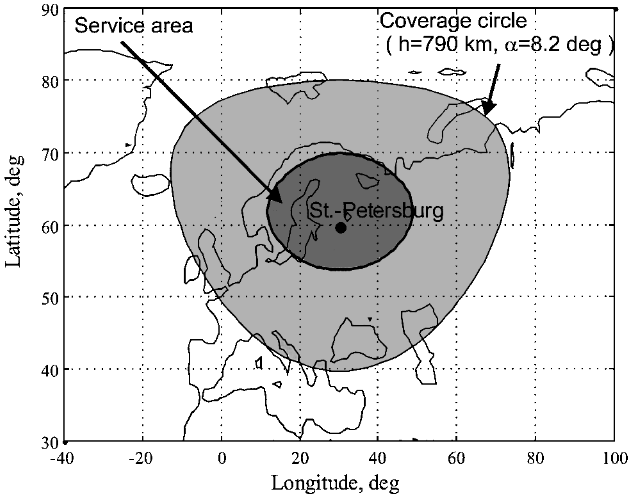


Fig. 10 Service area of type U–S–U for the IRIDIUM constellation.

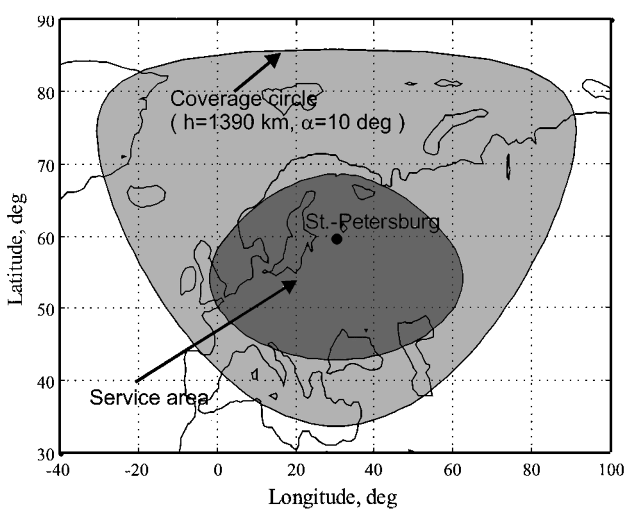


Fig. 11 Service area of type U–S–U for the GLOBALSTAR constellation.

$$\lambda_2 = \lambda_1 + \tan^{-1}\left(\sin\vartheta\sin\psi, \frac{\cos\vartheta - \sin\varphi_2\sin\varphi_1}{\cos\varphi_1}\right) \tag{16b}$$

As a rule, to obtain the converged values (to 0.5–1%), only three or four iterations are needed in this computation.

Figures 10 and 11 represent local service areas (U–S–U) of a user in St. Petersburg for the IRIDIUM and GLOBALSTAR constellations. The coverage circles of the user are shown for a comparison. The maximal link distances for azimuths of 0 and π, that is, south–north and north–south directions, are shown in Figs. 12 and 13. The IRIDIUM constellation supports the link (in a geometrical sense) between two users with a distance greater than 500 km for latitudes $\varphi \geq 40°$. In the case of the GLOBALSTAR constellation, this link can be established for two users near the equator with a distance no less than 400–1000 km and for middle latitudes with a distance no less than 1000–2000 km.

Regional and Global Communication Areas

The illustrated approach can be extended to a more complex scenario. The necessary condition of a link type S–GS–S between two users is a joint visibility of two satellites from the users and visibility of the two satellites from a GS. The computation of service area of type S–GS–S based on Eqs. (13–16) can be applied to the determination of a service area of type U–S–U. The communication between the users located in this service area can be established at any time. The summarized service area of a user, located in a service area of a GS, is a union of a service area of type U–S–U and a service area of GS(S–GS–S). An example of combined service area of a user in Berlin using a GS in Moscow based on the GLOBALSTAR constellation is shown in Fig. 14.

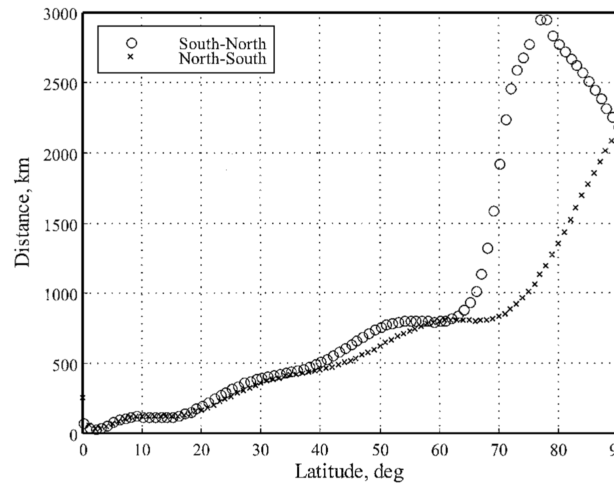


Fig. 12 Extremal link distances for the IRIDIUM constellation.

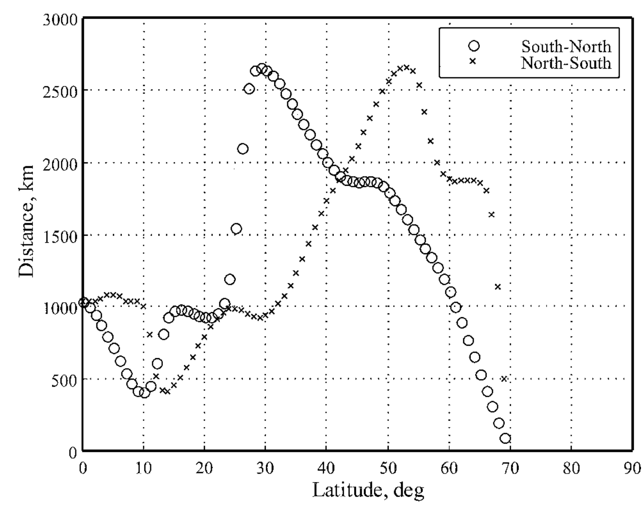


Fig. 13 Extremal link distances for the GLOBALSTAR constellation.

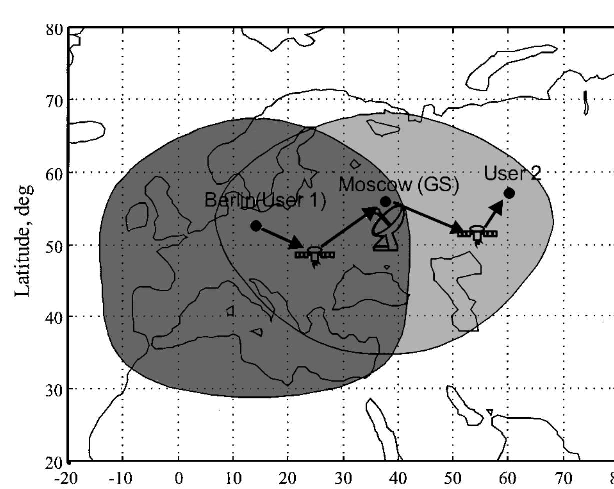


Fig. 14 Service area of user in Berlin using GS in Moscow.

Regional or global service areas can be described in terms of graph theory. Let us suppose that there is a GS network. Corresponding to each GS there are local service areas of type S– GS–S. A graph can be associated with the GS network. The vertices and edges correspond to the GSs and the pairs of ith and jth ground stations for which the link of type GS–S–GS is possible. The link between two GSs

Table 2 Characteristics of GS network

GS_i	GS_j	ϑ_{ij}, deg	Δt_1, min	δt_{12}, min
Berlin	Moscow	14.0	4.0	0.7
Berlin	Istanbul	14.9	6.5	1.9
Moscow	Istanbul	15.6	5.5	1.3
Moscow	Baku	18.0	4.0	0.7
Istanbul	Baku	16.3	6.4	2.0
Moscow	Ufa	10.3	6.5	1.6
Baku	Tashkent	14.4	7.4	2.5
Ufa	Tashkent	16.9	4.7	1.0
Ufa	Omsk	9.9	6.9	1.9
Omsk	Tashkent	14.3	6.6	1.9
Omsk	Krasnojarsk	11.1	5.7	1.3
Krasnojarsk	Chita	12.1	5.7	1.3
Chita	Komsomolsk-on-Amur	14.9	4.5	0.9
Komsomolsk-on-Amur	Petropavlovsk-Kamchatskiy	13.3	5.6	1.3

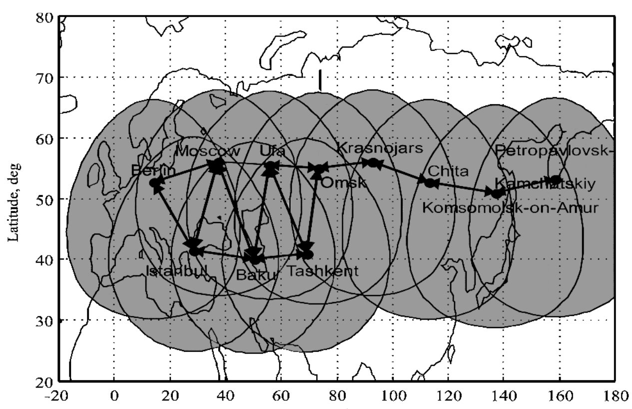


Fig. 15 Example of regional service area for the GLOBALSTAR constellation.

is similar to the local link between two users. The Earth-central angle between ith GS and jth GS ϑ_{ij}, the azimuth of connecting line Ψ_{ij}, and the coordinates of a middle point S_{ij} between them can be computed for given coordinates of the GSs (Fig. 9). Suppose that the θ_{GS} is the coverage angle of each GS. If

$$\vartheta_{ij} \leq 2\theta_{GS} - 2\theta_1(\varphi_{Sij}) \tag{17}$$

then, at any time, the ith and jth GSs see at least one satellite and the link between them can always be established. Similarly, the circle θ_S can be represented as the coverage circle of a user with the estimations for Δt_1, δt_{12}, and Δt_{1W}. The summarized service area of a LEO satellite communication system based on the GS network is a union of the service areas of all the GSs:

$$Z = Z_{GS1} \bigcup Z_{GS2} \bigcup Z_{GS3} \bigcup \cdots \bigcup Z_{GSn} \tag{18}$$

if the graph is connected.

As an example, a service area of some regions of Europe and Asia using the GLOBALSTAR constellation is shown in Fig. 15. A GS network comprises 11 GSs in Berlin (Germany), Moscow, Ufa, Omsk, Krasnojarsk, Chita, Komsomolsk-on-Amur, Petropavlovsk-Kamchatskiy (Russia), Istanbul (Turkey), Baku (Azerbaijan), and Taskent (Uzbekistan). The double pointers denote the pairs of continuously connected GSs. Parameters of this GS network (ϑ_{ij}, Δt_1, and δt_{12}) are given in Table 2.

Conclusions

We propose a novel geometric analysis method that includes both new statistical parameters of coverage for LEO satellite constellations and new algorithms for obtaining service areas. The numerical examples show that the algorithms described can be quite useful. We believe that, in a sense, the covering functions give a more descriptive geographical presentation than traditional coverage statistics based on an elevation angle.

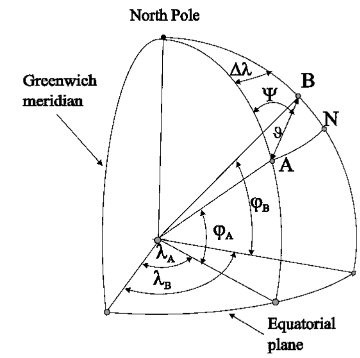


Fig. A1 Geographical coordinate frame.

The proposed methods may be useful for further research. First, the estimations of statistical parameters of coverage for LEO constellations can be used for a GS network design and combinatorial algorithms of link planning. Second, well-known methods of a satellite constellation synthesis[4–18] are oriented, as a rule, on a guarantee of a specified degree of coverage. It is evident from the foregoing that the synthesis problem of a LEO satellite constellation for communication systems is more complex than the guarantee of the degree of coverage. Perhaps special methods of satellite constellation synthesis with simultaneous optimization of the ground segment are required.

Appendix: Equations for Latitude and Longitude

For a given point A (latitude φ_A and longitude λ_A), Earth-centered angle ϑ, and azimuth Ψ, the geographical coordinates of the second point can be computed either from standard equations of spherical trigonometry. A simple spherical trigonometry procedure is shown in Fig. A1. The equations for the geographical latitude and longitude of the second point B, computed from spherical triangle ANB,[27] are

$$\varphi_B = \sin^{-1}(\sin\varphi_A \cos\vartheta + \cos\varphi_A \sin\vartheta \cos\Psi) \tag{A1}$$

$$\lambda_B = \lambda_A + \tan^{-1}\left(\sin\vartheta \sin\Psi, \frac{\cos\vartheta - \sin\varphi_B \sin\varphi_A}{\cos\varphi_A}\right) \tag{A2}$$

The inverse transformation for Earth-centered angle and azimuth between the two points A and B can be written as

$$\vartheta = \cos^{-1}[\sin\varphi_A \sin\varphi_B + \cos\varphi_A \cos\varphi_B \cos(\lambda_B - \lambda_A)] \tag{A3}$$

$$\Psi = \tan^{-1}\left(\sin(\lambda_B - \lambda_A)\cos\varphi_B, \frac{\sin\varphi_B - \sin\varphi_A \cos\vartheta}{\cos\varphi_A}\right) \tag{A4}$$

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